

How do quantum numbers generally vary in the adiabatic transformation of an ideal gas?

T. Yarman^{a)} and A. L. Kholmetski^{b)†}

^{a)}Department of Engineering, Okan University, Akfirat, Istanbul, Turkey & Savronik, Eskisehir, Turkey

^{b)}Department of Physics, Belarus State University, 4 Nezavisimosti Avenue 220030, Minsk, Belarus

(Received 3 March 2011; revised manuscript received 4 April 2011)

We continue to analyse the known law of adiabatic transformation for an ideal gas $PV^{5/3} = \text{Constant}$, where P is the pressure and V is the volume, and following the approach of non-relativistic quantum mechanics which we suggested in a previous work (Yarman *et al.* 2010 *Int. J. Phys. Sci.* **5** 1524). We explicitly determine the constant for the general parallelepiped geometry of a container. We also disclose how the quantum numbers associated with molecules of an ideal gas vary through an arbitrary adiabatic transformation. Physical implications of the results obtained are discussed.

Keywords: ideal gas, adiabatic transformation, non-relativistic quantum mechanics

PACS: 51.30.+i

DOI: 10.1088/1674-1056/20/10/105101

1. Introduction

It is known that classical thermodynamics operate in a useful approximation of an “ideal gas”, where it is thought that the molecules that comprise the gas do not interact with each other. Furthermore, the vibrational or rotational energy levels of molecules are not excited (see, e.g. Ref. [1]). In effect, any molecule is considered as being just in its translational motion in a given container.

The classical phenomenological laws for the description of an ideal gas were established long ago.^[1–4] In the present paper we focus on one of those laws

$$PV^\gamma = \text{Constant}, \quad (1)$$

which describes the adiabatic transformation of an ideal gas. Here P is the pressure, V is the volume and γ is $5/3$.

In our previous paper,^[5] we have shown that for a gas confined in a cube, undergoing a uniform adiabatic expansion, the constant on the right-hand side of Eq. (1), which seems to have never been questioned before, can easily be derived through non-relativistic quantum mechanics. It was thereby proven that this constant appears to be nailed to the square of the Planck constant, being inversely proportional to the mass of the molecule of which the gas is comprised. Here we should remember that the kinetic theory of

gas fails to disclose the value of the constant of Eq. (1) (see Ref. [2]).

In Section 2 we summarize briefly the framework of our approach. Then in Section 3 we reconsider the same problem, but with an ideal gas confined in a parallelepiped geometry, undergoing an arbitrary expansion, in which case the constancy of PV^γ within the framework of the quantum mechanical approach happens to be not so easy to prove. Nevertheless, we succeed in pinning down how quantum numbers associated with molecules of an ideal gas confined in parallelepiped geometry vary through an arbitrary adiabatic transformation, thus providing specifically the constancy of the product PV^γ . In Section 4 we briefly discuss the results obtained. It is important to note that we exclude all possible interactions of the constituents of the gas with each other, just as in the ideal gas approach of the kinetic theory of gases. Molecular excitation is further avoided.

2. Adiabatic transformation of ideal gas: quantum mechanical approach

It is known that the law of adiabatic transformation of an ideal gas, expressed by Eq. (1), is obtained in a familiar way (e.g., in Refs. [1] and [3]) from the

[†]Corresponding author. E-mail: khol123@yahoo.com

classical law of gases $PV = RT$ (written for one mole of gas) and the law of conservation of energy. Here, $R = 8.31 \text{ J/K}$ is the gas constant and T is the temperature expressed in K. The power γ is given by the ratio

$$\gamma = C_P/C_V, \quad (2)$$

where C_P is the heat to be delivered to one mole of ideal gas at constant pressure to increase the gas temperature by 1 K and C_V is the heat to be delivered to one mole of ideal gas at a constant volume to increase the gas temperature by 1 K.

For an ideal gas the internal energy levels of the constituents are not electronically excited by definition. Just their translational energies can be varied. In such a case^[1]

$$C_V = 3R/2, \quad (3)$$

$$C_P = 5R/2, \quad (4)$$

and the γ appearing in Eq. (1) turns to be equal to

$$\gamma = 5/3. \quad (5)$$

Hence equation (1), as is well known, acquires the form

$$PV^{5/3} = \text{Constant}. \quad (6)$$

This relationship can be tapped classically in just a few lines, for a set of states the gas can become transformed adiabatically from an initial state to a final one.^[2] Hence, the quantity PV^γ remains constant, throughout. However, it seems that it has never been considered what the specific alphanumeric expression of this constant may be, if one actually exists.

One usually is satisfied by writing $P_1V_1^{5/3} = P_2V_2^{5/3} = P_iV_i^{5/3}$ for adiabatic transformation between states 1, 2, and i , without wondering whether all of these quantities will in fact become equal to a particular constant.

Previously, Yarman *et al.*^[5] obtained Eq. (6) for an ideal gas based on non-relativistic quantum mechanics and provided an explicit expression for the constant of on the right-hand side of Eq. (6). It should be noted that their approach can be easily generalized to the relativistic case.

Based on the approach of Ref. [5] the authors succeeded in showing that the constant on the right-hand side of Eq. (6) for one mole of gas confined in a cube amounts to $N_A h^2 n^2 / (4m)$, where N_A is the Avogadro number, h the Planck constant, m the mass of the

molecule that the gas comprises and n the quantum number associated with the average molecule in all three directions.

Now we come to show why the quantum numbers associated with a given molecule remain the same within the cube and expand them uniformly.

To further simplify the problem, we consider a hypothetical one-dimensional box containing only one molecule, which moves back and forth along a given allowed single dimension. If this box expands adiabatically, then the expansion work will be done by the molecule of concern. The translational energy of the molecule will be reduced by the same amount. However, since the expansion is adiabatic, it becomes evident that the quantum mechanical energy level of the molecule will not be changed. In other words, the translational energy of the molecule within the box decreases, but the molecule still resides at the same quantum level.

Now, likewise, it is evident that when we transpose the one-dimensional adiabatic expansion into a three-dimensional adiabatic and uniform expansion of a cube, the one and the same quantum number associated with a given molecule will not be changed. The above may be condensed into the following assertion.

Assertion 1 The quantum number associated with a molecule confined in a one-dimensional hypothetical box expanding adiabatically is not changed through the expansion. Likewise, the one and the same quantum number associated with a given molecule, confined in a cube, remain the same, if the cube is expanded uniformly and adiabatically.

It is worth while to emphasize the common definition of an ideal gas that is comprised of strictly non-interacting molecules. The molecules, of course, interact with the walls of the recipient confining them. However, they do not interact with each other whatsoever. Thus each molecule behaves as a single quantum mechanical particle locked up (potential energy wise) in an infinitely high box. The wave functions of such non-interacting molecules are not mixed and thus the overall internal energy of the ideal gas can be found as a simple sum of energy eigenvalues tapped for all of the existing molecules.

Accordingly, we can state that we should not allow any irreversible process.

However, the average principal quantum number for any given molecule confined in a container of

macroscopic size can be very large near room temperature, but this does not create any conceptual difficulty. On the contrary, this allows us to neglect the exchange interaction (thus omitting the spin effects^[5]) and to use the classical kinetic theory of gases to characterize the macroscopic parameters of an ideal gas such as the pressure.

3. Ideal gas contained in parallelepiped geometry, undergoing a non-uniform expansion

The authors of Ref. [5] previously restricted their analysis to an ideal gas confined in a cube. The generalization to the case of parallelepiped container is not so trivial and requires more consideration.

We thus consider a particle of mass m at a fixed internal energy state located in a macroscopic parallelepiped of sides L_x , L_y , and L_z . Herein we will consider the non-relativistic case. However, the approach can be easily extended to the relativistic case.

The non-relativistic Schrödinger equation gives the energy E_n of the particle in the box at a given energy level^[6]

$$E_n = \frac{h^2}{8m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right), \quad (7)$$

where n_x , n_y , and n_z are the quantum numbers to be associated with the corresponding wave function dependencies on the relevant directions x , y , and z . For brevity, we introduce the subscript “ n ” which denotes the specific state characterized by the given set of integer numbers n_x , n_y , and n_z .

For non-interacting particles of an ideal gas, the potential energy within the box is null. Thus in the non-relativistic case we have

$$E_n = mv_n^2/2 \quad (8)$$

with v_n^2 being the averaged squared velocity of the particle at the n -th energy level.

At the given energy level, the pressure p_n exerted by just one particle on either wall can be written as

$$p_n = \frac{mv_n^2}{3L_xL_yL_z} = \frac{2}{3} \frac{E_n}{L_xL_yL_z} = \frac{2}{3} \frac{E_n}{V}. \quad (9)$$

For the sake of completeness, we recall the classical derivation of this equation. The force f_x exerted

by a molecule of mass m and velocity $\mathbf{v}_n\{v_x, v_y, v_z\}$ on the wall in the plane yz , is given by Newton’s second law $f_x = -\Delta p_x/\Delta t_x$, where $\Delta p_x = -2mv_x$ is the algebraic increment in momentum, whilst the molecule bounces back from the wall, $\Delta t_x = 2L_x/v_x$. Thus one obtains $f_x = mv_x^2/L_x$ and the pressure exerted by the molecule on the wall of concern becomes $p = f_x/(L_yL_z) = mv_x^2/V$. At the equilibrium state characterized evidently by the same temperature, one can write $v_x^2 = v_y^2 = v_z^2 = v_n^2/3$. Hence we arrive at Eq. (9).

Equations (7), (8), and (9), via the gas law $p_nV = kT$ written now for just one molecule (k being the Boltzmann constant), allow us to write

$$\begin{aligned} E_n &= \frac{h^2}{8m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right) \\ &= \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{3}{2}kT, \end{aligned} \quad (10)$$

and at the given stationary state delineated by the temperature T , one can conjecture that

$$\begin{aligned} \frac{h^2}{8m} \frac{n_x^2}{L_x^2} &= \frac{1}{2}mv_x^2 = \frac{h^2}{8m} \frac{n_y^2}{L_y^2} \\ &= \frac{1}{2}mv_y^2 = \frac{h^2}{8m} \frac{n_z^2}{L_z^2} = \frac{1}{2}mv_z^2, \end{aligned} \quad (11)$$

or, in short

$$\frac{n_x}{L_x} = \frac{n_y}{L_y} = \frac{n_z}{L_z}. \quad (12)$$

This happens to be the equilibrium condition for the parallelepiped geometry. This occurrence deserves to be stated as our next assertion.

Assertion 2 In a parallelepiped geometry, built on sizes L_x , L_y , and L_z , and containing a given molecule at a given quantum state n described by the integers n_x , n_y , and n_z at the equilibrium, one has $n_x/L_x = n_y/L_y = n_z/L_z$.

Combining now Eqs. (7) and (9), we calculate (for just one particle at the quantum level n) the product $p_nV^{5/3}$ as

$$\begin{aligned} p_nV^{5/3} &= \frac{2}{3} \frac{(h^2/8m) (n_x^2/L_x^2 + n_y^2/L_y^2 + n_z^2/L_z^2)}{V} V^{5/3} \\ &= \frac{h^2}{12m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right) (L_xL_yL_z)^{2/3}. \end{aligned} \quad (13)$$

Normally we have many particles (rather than just one particle) at different quantized states. This means that we deal with the energy distribution of molecules at a given temperature, instead of the fixed eigenvalue given by Eq. (7) derived for a gas composed of a single molecule.

Just as the authors of Ref. [5] did before, in order to describe the distribution of energy over the molecules within an elaborated quantum mechanical approach, we have to abandon the strict ideal gas approximation and introduce into the Schrödinger equation a potential energy term as a perturbation, which would be then responsible for the weak interaction of molecules, whose averaged value will somewhat randomly affect all of the molecules of gas at each fixed temperature.

We recall that such a distribution of perturbation energy is to be compatible with the Maxwellian distribution of velocities. The analysis of this problem though falls outside the scope of the present study.

Thus, for our immediate purpose it is sufficient to take into consideration an “average molecule” at the given temperature T . We can visualize the average molecule as a single particle obeying Eq. (13), thus it is situated at the given n -th level, therefore relating the given temperature to the energy of concern.

For one mole of ideal gas, which is thus composed of N_A molecules, we can easily generalize Eq. (13) for the macroscopic pressure P_n exerted at the given average state n by one mole of gas on the walls of the container, and the resulting equation is expressed as

$$P_n V^{5/3} = \frac{h^2}{12m} \sum_{i=1}^{N_A} \left(\frac{n_{xi}^2}{L_x^2} + \frac{n_{yi}^2}{L_y^2} + \frac{n_{zi}^2}{L_z^2} \right) \times (L_x L_y L_z)^{2/3}, \quad (14)$$

where the quantum numbers n_{ix} , n_{iy} , n_{iz} are associated with the i -th molecule.

Equation (14) shows that for a cubic container ($L_x = L_y = L_z$) $P_n V^{5/3}$ indeed turns out to be a constant. However, for a parallelepiped geometry ($L_x \neq L_y \neq L_z$) the constancy of $P_n V^{5/3}$ is not so obvious. More specifically, if one assumes, as a rough approach, the constancy of quantum numbers n_{ix} , n_{iy} , and n_{iz} under the adiabatic transformation of an ideal gas, just the way this takes place, in effect, in a “one-dimensional” box (cf. Assertion 1), where only one quantum number comes into play with the given molecule; the right-hand side of Eq. (14) then becomes dependent on the size of container, which evidently contradicts the classical result $PV^{5/3} = \text{Constant}$.

Therefore one can see that, in general, in the three-dimensional case the quantum numbers n_{ix} , n_{iy} , and n_{iz} must be changed through an adiabatic transformation of the ideal gas. The exception is the case

where the adiabatic transformation is achieved uniformly (i.e. $L_x \rightarrow fL_x$, $L_y \rightarrow fL_y$, and $L_z \rightarrow fL_z$; f being a multiplier characterizing the expansion), and the right-hand side of Eq. (14) remains constant for the fixed set of quantum numbers n_{ix} , n_{iy} , and n_{iz} .

Therefore, we come to the following assertion.

Assertion 3 The quantum numbers associated with a given molecule confined in a parallelepiped are not changed through a uniform adiabatic expansion of the box.

From the physical viewpoint this result can be understood via the observation that the uniform adiabatic expansion of a container keeps the equilibrium condition (12) untouched. In contrast, any non-uniform adiabatic transformation of the container breaks the original equilibrium, i.e. Eq. (12), and leads to a new equilibrium, achieved after multiple collisions of the molecules with the walls of the container (we recall that in an ideal gas the molecules are assumed not to collide with each other). In such a case the quantum numbers n_x , n_y , and n_z are, in general, changed (here, we have omitted the subscript “ i ” for brevity).

In order to determine the law of transformation of quantum numbers, we reconsider the uniform adiabatic expansion of a parallelepiped container, implemented through the following three stages.

(i) An adiabatic expansion in the x direction only: $L_x, L_y L_z \rightarrow fL_x, L_y L_z$ (where $f > 1$ for the expansion process);

(ii) A further adiabatic transformation in just the y direction: $fL_x, L_y L_z \rightarrow fL_x, fL_y L_z$;

(iii) A final adiabatic transformation in just the z direction: $fL_x, fL_y L_z \rightarrow fL_x, fL_y fL_z$.

Next, we assume that the quantum number n_x at the first stage is modified by ε_{xx} , at the second stage by a further coefficient ε_{xy} and at the third stage by a final coefficient ε_{xz} , so that at this latter stage we arrive at the overall transformation $n_x \rightarrow \varepsilon_{xz}\varepsilon_{xy}\varepsilon_{xx}n_x$. Similar coefficients are to be introduced for the quantum numbers n_y and n_z . For example, at the first stage we have the transformation $n_y \rightarrow \varepsilon_{yx}n_y$: at the second stage we have the transformation $\varepsilon_{yx}n_y \rightarrow \varepsilon_{yy}\varepsilon_{yx}n_y$, and at the third stage we arrive at the transformation $\varepsilon_{yy}\varepsilon_{yx}n_y \rightarrow \varepsilon_{yz}\varepsilon_{yy}\varepsilon_{yx}n_y$.

Notice that after the implementation of all these stages, on the whole, we obtain a uniform adiabatic expansion of the container, where the quantum numbers n_x , n_y , and n_z become equal to their original values. Hence we arrive at the equalities as follows:

$$\varepsilon_{xx}\varepsilon_{xy}\varepsilon_{xz} = 1, \quad \varepsilon_{yx}\varepsilon_{yy}\varepsilon_{yz} = 1, \quad \varepsilon_{zx}\varepsilon_{zy}\varepsilon_{zz} = 1, \quad (15)$$

which, in combination with Eq. (12), yield¹⁾

$$\varepsilon_{ll} = f^{2/3}, \quad \varepsilon_{l \neq m} = f^{-1/3} \quad (l, m = x, y, z). \quad (16)$$

Equations (15) and (16) disclose the law of transformation of quantum numbers

$$n_l(f) \rightarrow \varepsilon_{lm}(f) n_l, \quad (17)$$

for the adiabatic expansion ($f > 1$) or adiabatic compression ($f < 1$) of the container along the dimension m by the coefficient f .²⁾ We state this occurrence as our next assertion.

Assertion 4 Consider a molecule imprisoned in a parallelepiped of sizes L_x , L_y , and L_z . The molecule resides at a given quantum state n described by the integers n_x , n_y , and n_z . If the box expands adiabatically along the x direction, so that $L_x \rightarrow fL_x$, then the quantum numbers become transformed as follows: $n_x \rightarrow f^{2/3}n_x$, $n_y \rightarrow f^{-1/3}n_y$, and $n_z \rightarrow f^{-1/3}n_z$.

Therefore, for the adiabatic transformation of the container along the x axis ($L_x, L_yL_z \rightarrow fL_x, L_yL_z$), via combining Eqs. (14), (16), and (17), we obtain

$$\begin{aligned} P_n V^{5/3} &= \frac{\hbar^2}{12m} \sum_{i=1}^{N_A} \left(\frac{n_{xi}^2 f^{4/3}}{f^2 L_x^2} + \frac{n_{yi}^2 f^{-2/3}}{L_y^2} \right. \\ &\quad \left. + \frac{n_{zi}^2 f^{-2/3}}{L_z^2} \right) f^{2/3} (L_x L_y L_z)^{2/3} \\ &= \frac{\hbar^2}{12m} \sum_{i=1}^{N_A} \left(\frac{n_{xi}^2}{L_x^2} + \frac{n_{yi}^2}{L_y^2} + \frac{n_{zi}^2}{L_z^2} \right) \\ &\quad \times (L_x L_y L_z)^{2/3}, \end{aligned} \quad (18)$$

which now indeed shows the constancy of the product $PV^{5/3}$ through a non-uniform adiabatic transformation of the gas at hand, confined in a parallel pipe.

Analogously, one can derive the constancy of $PV^{5/3}$ for the transformation of the size of a container along the y and z axes.

Further, we should interpret the quantum numbers n_{xi} , n_{yi} , and n_{zi} , and the sizes L_x , L_y , and L_z on the right-hand side of Eq. (18) as the initial parameters of the system “the ideal gas confined in the container” at a given initial temperature T_0 . Thus the constant of the adiabatic transformation can be

¹⁾ Here we should remember that both the initial and the final sets of quantum numbers must appear as integers in order to maintain the boundary condition of the vanished wave function on the walls of the container. This means that each of the coefficients ε_{lm} must not appear as a continuous function. However, the quantum numbers are so huge for an ideal gas confined in a macroscopic container near the room temperature^[5] and thus, we may overlook the mentioned constraint of non-continuity of the coefficients ε_{lm} .

²⁾The general case of a non-uniform transformation of a container in all three dimensions simultaneously can be processed as a succession of corresponding non-uniform transformations in each dimension by using Eq. (17).

written in the form

$$\begin{aligned} \text{Constant} &= \frac{\hbar^2}{12m} \sum_{i=1}^{N_A} \left(\frac{n_{xi}^2(T_0)}{L_x^2(T_0)} + \frac{n_{yi}^2(T_0)}{L_y^2(T_0)} \right. \\ &\quad \left. + \frac{n_{zi}^2(T_0)}{L_z^2(T_0)} \right) (V(T_0))^{2/3}, \end{aligned} \quad (19)$$

and any further adiabatic transformation of an ideal gas keeps the constancy of $PV^{5/3}$ in full agreement with the classical result.

Equation (19) fulfills our goal in this paper. We have indeed come to obtain not only the classical adiabatic relationship $PV^\gamma = \text{Constant}$ in a parallelepiped geometry via non-relativistic quantum mechanics, which we geared to the classical definition of ideal gas, but also the specific value of the constant on the right-hand side of Eq. (19); furthermore we have disclosed the rule about the modification of quantum numbers of molecules, expressed via Eqs. (16) and (17).

For the special case of a cube, the right-hand side of Eq. (19) becomes

$$\begin{aligned} \text{Constant}_{\text{cubic container}} &= \frac{\hbar^2}{4m} \sum_{i=1}^{N_A} n_{ix}(T_0)^2 \\ &= N_A \frac{\hbar^2 n(T_0)^2}{4m}, \end{aligned} \quad (20)$$

where n pertains to the average molecule as stated in the previous work.^[5]

Therefore, our result indicates that the behaviour of an ideal gas can be understood within a quantum mechanical approach and the constancy of PV^γ happens to be rooted in quantum mechanics.

Note further that Eq. (19) holds for a photon gas and we spare the discussion of further interesting points for a future work.

4. Discussion

Thus, having obtained the law of adiabatic transformation for an ideal gas, i.e. Eq. (1) from non-relativistic quantum mechanics, we not only find a new way to derive this law, but also obtain the value

of the constant appearing in the classical expression of the constancy of the product $PV^{5/3}$ (see Eq. (19)).

We further determine the law of transformation of principal quantum numbers (Eqs. (16) and (17)) under a non-uniform adiabatic expansion of a parallelepiped container.

The constant on the right-hand side of Eq. (19) happens to be inversely proportional to the mass m of the molecule that the gas comprises. It seems useful to carry out a cross-check exercise, based on this phenomenological law written in the following form:

$$PV^{5/3} = \frac{\text{constant}'}{m} = \text{constant}, \quad (21)$$

where the constant' (cf. Eqs. (19) and (20)) has the dimension of the square of the Planck constant.

Therefore, one can achieve the following outcomes. Consider different ideal gases composed of different molecules with respective masses m_1, m_2, m_i , etc., contained in (for simplicity, though without any loss of generality) different cubes of equal size. Suppose that each gas amounts to one mole. They are at respective temperatures T_1, T_2, T_i , etc., and we have $m_1T_1 = m_2T_2 = m_iT_i$, so that one can come out with the same average quantum number for all of them, at the given initial temperatures T_1, T_2, T_i (cf. Eq. (10)).

Now, we measure the pressure and calculate for each gas the product $P_iV_i^{5/3}$, then plot this quantity versus $1/m_i$. One should then obtain an increasing straight line and the slope of it must turn out to be nailed to h^2 .

Thus, if experiments with rarefied gases, which are well described by the ideal gas approximation, confirm the validity of Eq. (20) (and we do not have any reasons to doubt it), then such a result definitely indicates that the relationship between the classical law of gas and quantum mechanics holds true.

It is further interesting to comment on the law of transformation of principal quantum numbers for an adiabatic expansion (or compression) of an ideal gas (Eqs. (16) and (17)), based on the known form of the wave function associated with a molecule,^[7] i.e.

$$\begin{aligned} \psi(x, y, z) = & \sqrt{\frac{8}{L_x L_y L_z}} \sin \frac{\pi n_x}{L_x} x \\ & \times \sin \frac{\pi n_y}{L_y} y \sin \frac{\pi n_z}{L_z} z. \end{aligned} \quad (22)$$

The boundary conditions for an infinitely high box, where its left bottom corner is located at the origin of

coordinates, are

$$\begin{aligned} \psi(0, y, z) = \psi(x, 0, z) = \psi(x, y, 0) = \psi(L_x, y, z) \\ = \psi(x, L_y, z) = \psi(x, y, L_z) = 0. \end{aligned} \quad (23)$$

Further, we have found above that for the uniform adiabatic expansion of the box ($L_x, L_y L_z \rightarrow fL_x, fL_y, fL_z$) the quantum numbers n_x, n_y , and n_z remain unmodified. In view of Eq. (21), this means that the periods of the sine functions embodied by the wave function along each spatial dimension are also multiplied by the factor f . This observation conforms fully with the known result that the adiabatic expansion of an ideal gas leads to the decrease of the translational energy of its constituents, thereby causing their de Broglie wavelength to stretch correspondingly. At the same time, it is interesting to recall that in a non-uniform expansion of the container, say, in the case where L_x increases by the factor f , so that ($L_x, L_y L_z \rightarrow fL_x, L_y, L_z$); the spatial period of the wave function along the x direction, in effect, increases by a coefficient smaller than f , leading to an increase in quantum number n_x (see Eqs. (16) and (17) at $l, m = x$). Concurrently the overall decrease of the translational energy of a molecule under an adiabatic expansion of the ideal gas stipulates that the corresponding increase in the spatial period of the wave function along the y and the z directions, with sizes L_y and L_z fixed, leads to the decrease of quantum numbers n_y and n_z as indicated by Eqs. (16) and (17). In a similar way one can consider that the quantum numbers vary with the expansion of the ideal gas along other spatial dimensions.

We hope that the novel derivation of the law of adiabatic transformation of an ideal gas, which we have presented in the paper, will be useful for a better understanding of its relationship with quantum mechanics, and thus enriches its physical content.

Last but not the least, our derivation displays how the mass (mass of the molecules that the gas at hand comprises), space (size of the container at hand), and the time (period of time of the average molecule in consideration takes to cross the box of concern from one edge to another) must be organized, even in the very complicated case of a gas made of billions of particles. As shown in Refs. [8]–[12], the product of mass by pressure by volume to the power 5/3 must be nailed down to a universal constant, more precisely the square of the Planck constant, in order to cope

with the end results of the Special Theory of Relativity, which the box brought together to a uniform translational motion.

References

- [1] Sommerfeld A 1964 *Thermodynamics and Statistical Mechanics* (New York: Academic Press)
- [2] Halliday D, Resnick R and Walker J 1997 *Fundamentals of Physics* (New York: Wiley)
- [3] ter Haar D and Wergeland H 1966 *Elements of Thermodynamics* (Reading, MA: Addison-Wesley)
- [4] van Wylen G J and Sonntag R E 1985 *Fundamentals of Classical Thermodynamics* (New York: Wiley)
- [5] Yarman T, Kholmetskii A and Korfali O 2010 *Int. J. Phys. Sci.* **5** 1524
- [6] Moore W J 1955 *Physical Chemistry* (New York: Prentice Hall, Inc.)
- [7] Landau L D and Lifshitz E M 1979 *Quantum Mechanics: Non-relativistic Theory* 3rd edn. (Oxford: Elsevier Science)
- [8] Yarman T 2004 *Optics and Spectroscopy* **97** 683
- [9] Yarman T 2004 *Optics and Spectroscopy* **97** 691
- [10] Yarman T 2009 *Int. J. Theor. Phys.* **48** 2235
- [11] Yarman T, Kholmetskii A L and Korfali O 2011 *Journal of Thermal Science and Technology*, in press
- [12] Yarman T 2010 *Quantum Mechanical Framework Behind the End Results of the General Theory of Relativity — Matter Is Built on a Universal Matter Architecture* (New York: Nova Publishers)